Self-assessment exercises: Computational Electromagnetics

1 Basics

1. What is the definition of the electric current density and how is it linked to the electric field strength?
2. What is the definition of the Coulomb's law of two electrical point charges Q_1 and Q_2 in free space with a distance r?
3. What is the Lorentz force of a moving point charge Q in an electromagnetic field?
4. What is the difference of the electric potential and voltage?
5. What is the amount of mechanical work performed, if a charge Q is moved along a curve C in an electromagnetic field (no repercussion on the electric field). Does the force of the magnetic field contribute to the performed work?
6. An accumulated charge Q moves along a metallic conductor with a volume of $V = L \cdot A$ (A : cross-section, L : length) in a magnetic field $\vec{B}$. What is the current in the conductor? State the Lorentz force in the metallic conductor in dependence of the current.
7. State the definition of the magneto-motive-force and the corresponding SI units.
8. What causes magnetic fields according to the Oersted's law?
9. What is the definition of the magnetic flux through face A?

2 Maxwell's Equations

1. State Maxwell's equations in global, differential and integral form.
2. Use Oersted's law for a plate capacitor attached to an AC source. Can you see a contradiction – how can it be solved?
3. Deduce the Poisson equation of electrostatics from Maxwell's equations.

4. Deduce the equation of continuity from Maxwell's equations.

5. How can Maxwell's equations be simplified for stationary flow fields?

3 Material – and Continuity Equations

1. State the general constitutive relations.

2. What is the definition of polarization and magnetization?

3. How can the interface conditions of the electrical flux density and magnetic field strength be derived from Maxwell's equations?

4. State the general condition for the electric field strength $\vec{E}$, the electric flux $\vec{D}$ and the potential ϕ at the interface of two materials with different properties.

5. How can the tangential component of the magnetic field/electric field be continuous across the interface of two materials?

6. How can the normal component of the electric flux density be continuous at the interface of two materials?

7. What are the boundary conditions of the electric field, the magnetic flux density and magnetic field at the interface to a perfect electric conductor?

4 Electrostatics

1. State the three-dimensional Greens function of electrostatics.

2. What is the energy density w_e of a point charge Q in vacuum as a function of Q and distance r?

3. The definition of the scalar potential ϕ is based on which Maxwell equation?

4. A metallic conductor is situated in an electrostatic field. How are the influence charges distributed? What is the value of the electric field strength inside of the conductor? What is the orientation of the electrostatic field at the surface?

5. On a hollow sphere a charge Q is distributed homogeneously. How is the electric field oriented in- and outside of the sphere?

6. Why is the electric potential ϕ constant inside of a charge free conductive hollow body?

7. Explain the method of image charges.

5 Exercises

1. Compute the electric field of a point charge Q where $\vec{E} = -\text{grad}\,\phi$ and $\phi = \frac{Q}{4\pi\epsilon_0 r},\ (r > 0)$.
2. Compute the divergence of the vector field $\vec{A} = (x^3y^2z, \sin(y)+y^2z, xz^2)^T$.
3. Compute the curl of the vector field $\vec{A} = (x^2y, yz^3, xyz)^T$.
4. Verify the relations
   - $\text{curl}\,\text{grad}\,\phi = 0$,
   - $\text{div}\,\text{curl}\,\vec{A} = 0$,
   - $\text{curl}\,\text{curl}\,\vec{A} = \text{grad}\,\text{div}\,\vec{A} - \Delta\vec{A}$.
5. Derive the PDEs describing electrostatic, magnetostatic, electroquasistaticand magnetoquasistatic field problems and the electromagnetic wave equation.