Self-Assessment exercise: Computational Fluid Dynamics (CFD)

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Question 1: Gasdynamics - compressible gas flow

Fig. 1 shows the converging-diverging nozzle of a rocket. The gas (gas constant $R = 287\,J/kgK$, ratio of specific heats $\kappa = 1.4$) flows from a huge vessel (pressure p_i, temperature T_i) through a nozzle into the atmosphere (pressure $p_o = 1\,bar$). The flow in the nozzle can be assumed steady-state and isotropic. The minimum cross section is $A_A = 1m^2$. The values for the pressure are absolute pressure values.

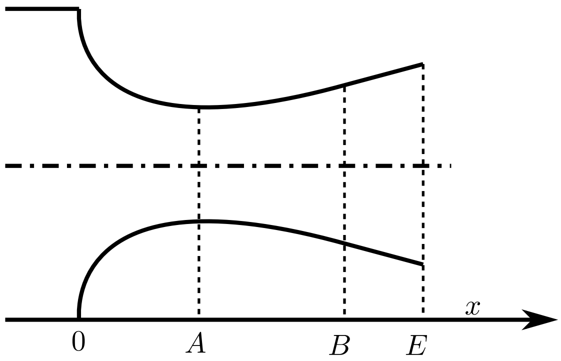

Figure 1: Flow of an compressible gas through a nozzle

The pressure in the vessel is $p_i = 5.2\,bar$ and the temperature $T_i = 300\,K$.

a) Calculate the mass flow through the nozzle

b) Calculate the thrust of the nozzle

c) Calculate the cross section A_E at the end of the nozzle assuming an ideal expansion of the gas

Due to a leakage in the vessel, the pressure in the vessel is $p_i = 1.5\,bar$ and the temperature $T_i = 300\,K$.

d) Calculate the mass flow through the nozzle

e) Calculate the thrust of the nozzle

During the start of the rocket the diverging part of the nozzle (beginning at position A) breaks away. The pressure in the vessel is $p_i = 5.2\,bar$ and the temperature $T_i = 300\,K$.

f) Calculate the thrust of the rocket.

Question 2.1: Navier-Stokes equations

Fig. 2 shows a rotational viscosimeter. The dynamics viscosity μ of an incompressible liquid (density ρ) can be measured by determining the torque of the rotating cylinder (rotational speed ω). The cylinder (length l) with an outer radius r_1 rotates in a cylindrical (non rotating, completely filled with the liquid) vessel with an outer radius r_2. The distance of the cylinder to the bottom of the vessel is b. The liquid can be described as a Newtonian liquid. The torque on the cylinder can be split up in two components - the torque in gap 1 (between the concentric cylinders) and the torque in gap 2 between the rotating cylinder and the bottom of the vessel.

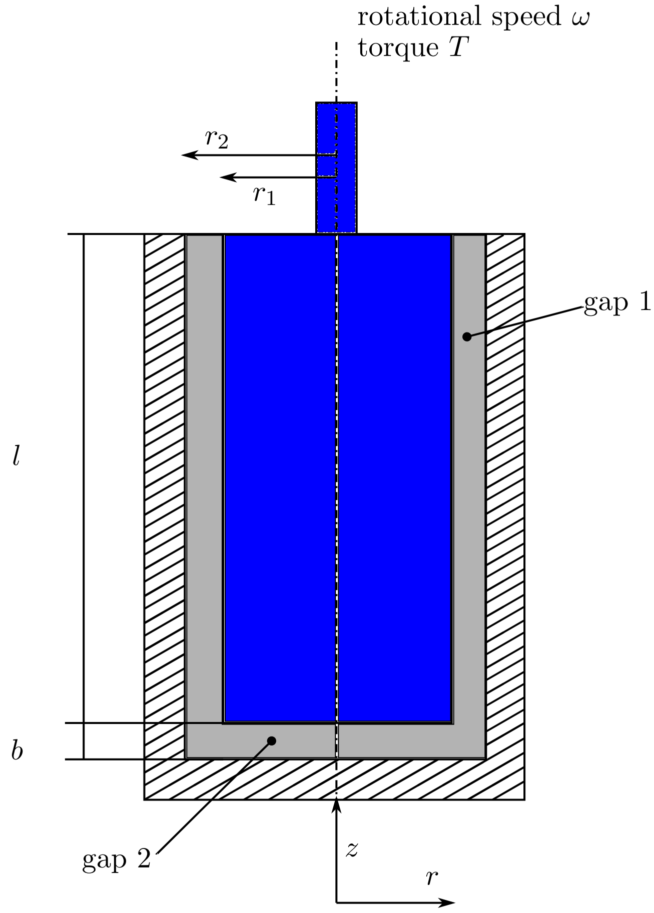

Figure 2: Drawing of the rotational viscosimeter

a) Determine the velocity of the liquid in gap 1 between the radius r_1 and r_2. Please state the assumptions you used for the derivation of the velocity.

b) Determine the torque T_1 in gap 1 as a function of the rotational speed ω, viscosity μ and the geometrical parameters.

c) How does the velocity profile in gap 2 look like. Make a schematic drawing of the velocity profile in gap 2. Are there any difference compared to part a). No calculations are required.

d) Describe the way to calculate the torque in gap 2. Are there any differences to the calculation in part b). No calculations are required.

Question 2.2: Navier-Stokes equations

Fig. 3 shows two moving belts which have a distance b. The velocity of the belts is determined by the radius r and the rotating speeds ω_1 and ω_2. Between the plates, a fluid (density ρ, dynamic viscosity μ) is driven by a pressure gradient $\frac{\partial p}{\partial x}$. The belts are very long, i.e. the influence of the inflow can be neglected. In z-direction, the depth will be t.

Pay attention to the direction of the velocities!

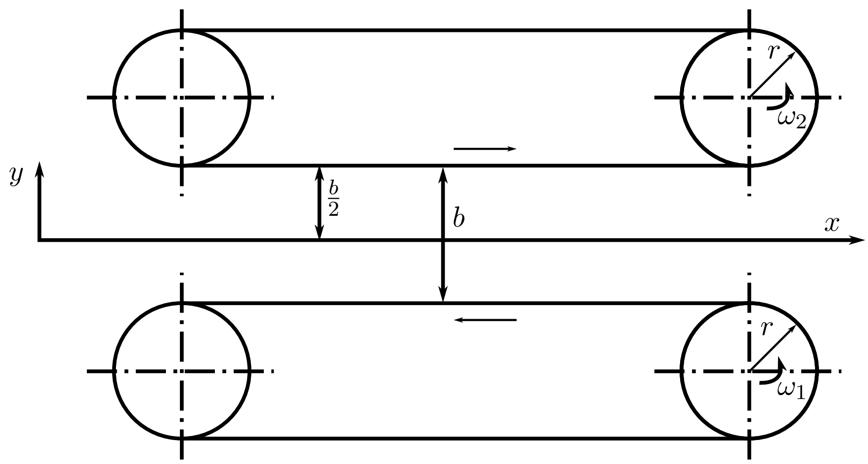

Figure 3: Drawing of the flow between to moving belts

a) Determine the velocity of the liquid in the gap between the belts. Derive the equation to calculate the velocity from the Navier-Stokes equation using appropriate assumptions and boundary conditions.

b) Make a schematic drawing of the velocity u in x-direction for three different cases:

- $\omega_1 = \omega_2 = 0$, $\frac{\partial p}{\partial x} < 0$
- $\omega_1 = \omega_2$, $\frac{\partial p}{\partial x} = 0$
- $\omega_1 = \omega_2$, $\frac{\partial p}{\partial x} < 0$

c) Calculate the volume flow Q in the gap as a function of the given parameters

d) The rotational speed ω_2 is equal zero. Calculate the rotational speed ω_1 for which a volume flow $Q = 0$ will be observed.

Question 3: Flow through a foam

Fig. 4 shows an experimental setup to determine the heat conduction in a foam. Gas with a temperature T_1 enters the experimental setup on the left side ($x = 0$) with a constant velocity u. After the distance l_1 the flow enters the foam (porosity $\epsilon = 0.5$) and leaves the foam after a distance l_2 with a temperature T_2. The experimental setup is adiabatic, i.e. there is no heat transfer to the surrounding parts. The temperature distribution can be assumed steady-state and is only dependent on the x-direction.

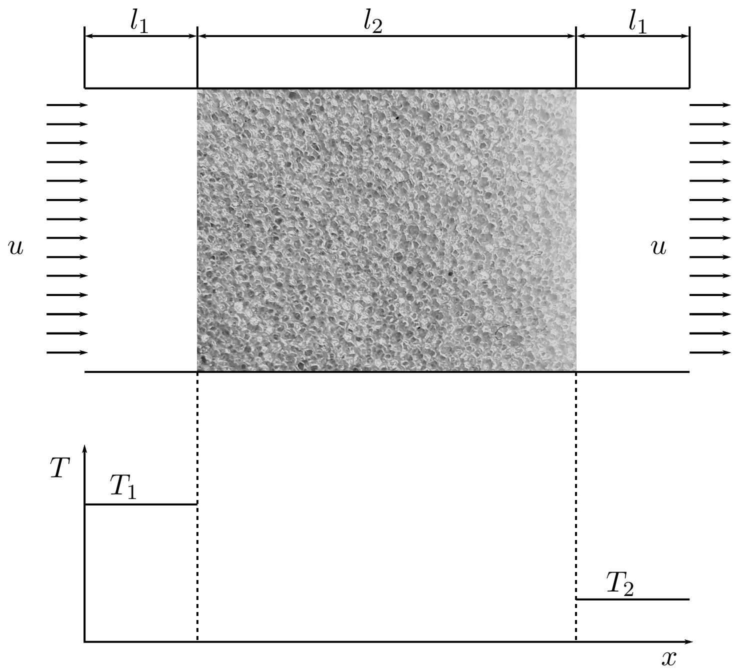

Figure 4: Flow through a foam

a) Derive the differential equation to calculate the temperature in the foam as a function of x and describe the assumptions made.

b) Solve the differential equation using a function

$$T(x) = C_1 + C_2 e^{\frac{\rho c_p u}{k} x}$$

and plot schematically the temperature distribution in Fig. 4. Plot another curve with a higher velocity u. How does the temperature profile look like?

c) How can we consider the porosity in the calculations?

Question 4: Questions

a) Give a definition of the sonic speed? What parameters influence the sonic speed?

b) Is the sonic speed of water higher or lower than the sonic speed of air (for ambient temperature and pressure)?

c) Plot the sonic speed of a two-phase flow (air / water) against the volume fraction of the liquid.

d) Why can't we increase the gas velocity for an compressible flow through a converging nozzle? What is the effect of an converging-diverging nozzle?

e) What is the effect of a shock in compressible flows? What happens?

f) Give a definition of the acceleration in fluids? What is the difference between structural mechanics and fluid mechanics?

g) Explain the meaning of continuum mechanics? What is the idea behind?

h) Can we use continuum mechanics for flows in vacuum? Why or why not?

i) Plot the drag coefficient of the flow past a circular cylinder. Describe the curve and explain the different regions of the curve

j) Plot the drag coefficient of the flow past a cube. Plot the velocity field around the cube for low and high velocities

k) What is the idea behind the Reynolds-averaged Navier-Stokes equations? Explain the method using the continuity balance for a 1D-flow.

l) What is the difference between forced and free convection heat transfer?

m) Does the velocity of gases increase or decrease with increasing temperatures? What happens for liquids?

n) Explain why dimensionless numbers are used in fluid mechanics? Name two dimensionless numbers in fluid mechanics.

o) Name three different mechanisms of heat transfer?

p) Explain the difference between the derivatives $\frac{Du}{Dt}$ and $\frac{\partial u}{\partial t}$. Explain the difference with an example from fluid mechanics.