Self-assessment exercise: Materials Science

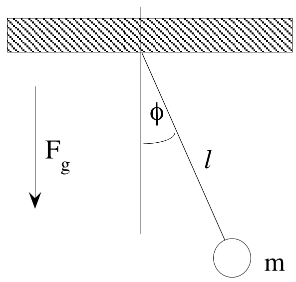
Volume work

A compressor adiabatically compresses a gas, which originally is at room temperature T_o and ambient pressure P_o. After the gas has passed through a water cooled system of pipes (isobaric cooling), it leaves the machine again possessing the temperature T_o but with the pressure P_1.

Calculate the work necessary for this process w_a. Also calculate the work w_b in the case of a reversible, isothermal compression yielding the identical final state. Sketch the ratio w_a/w_b as function of P_1/P_o with $C_V = (5/2)nR$. Hint: Make a sketch of the processes in term of suitable thermodynamic variables. Assume an ideal gas.

Thermal energy of a pendulum

(a) Calculate the average thermal energy $\langle E \rangle$ of the pendulum in the sketch in the limit of small oscillations ($\cos\phi \approx 1 - \phi^2/2$). Express your result in terms of the gravitational acceleration g and the length of the pendulum l. Notice that the 1D harmonic oscillator posses the energy eigenvalues $E_\nu = \hbar\omega(\nu + 1/2)$. It is ω which must be expressed in terms of the above quantities.



(b) Sketch $\langle E \rangle/(\hbar\omega)$ and the reduced heat capacity C_V/k_B vs. T/T_s, where T_s is a characteristic temperature of the pendulum. Estimate the range of temperatures in which the pendulum behaves classically. How long must the length of the pendulum be in order for it to behave classically at $T = 300K$?

Magnet

A simple model of the thermal behavior of a magnet has energy values $E_\nu = -Jm_\nu\langle m \rangle$, where the coupling constant $J > 0$. In addition $\nu = 1, 2$ and $m_\nu = \pm 1$. Derive an implicit formula for the average magnetization $\langle m \rangle$ in terms of T/T_c ($T_c = J/k_B$). Notice that for $T < T_c$ there is more than one solution. Which of these solutions are thermodynamically stable? Make a sketch of the stable magnetization(s) in the temperature range $0 < T < \infty$.