Self-assessment exercises: Theoretical Particle Physics

1. Consider a particle with electric charge q and mass m moving in the presence of an electromagnetic field characterized by the scalar and vector potentials V and $\vec{A}$. The Lagrangian for this system is given as

$$\mathcal{L} = \frac{1}{2}m(\vec{v}\cdot\vec{v}) + q(\vec{v}\cdot\vec{A}) - qV.$$

   - Find the equation of motion for the particle.
   - Apply a gauge transformation of the form

$$\vec{A} \to \vec{A} + \nabla\lambda(\vec{x},t)$$
$$V \to V - \frac{d}{dt}\lambda(\vec{x},t)$$

     - Write down the Lagrangian with the transformed potentials. Did it change? What about the equation of motion?
     - Using the definitions of $\vec{E}$ and $\vec{B}$ from the potentials

$$\vec{B} = \nabla\times\vec{A}$$
$$\vec{E} = -\nabla V - \frac{\partial}{\partial t}\vec{A}$$

       show that these fields are unaffected by such gauge transformations. How is this related to the gauge invariance of the equation of motion?

2. Consider a quantum harmonic oscillator. A coherent state $|\alpha\rangle$ is defined as

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle\,,$$

   where $\hat{a}$ is the annihilation operator and α a complex parameter.

   - Using the fact that one can write $|\alpha\rangle$ as a linear combination of energy eigenstates $|n\rangle$ and that these can be written as

$$|n\rangle = \frac{\left(\hat{a}^\dagger\right)^n}{\sqrt{n!}}\,|0\rangle$$

     show that a normalized coherent state can be written as

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}e^{\alpha\hat{a}^\dagger}\,|0\rangle\,.$$

- Find the average number of quanta ($\langle \hat{n} \rangle$ with $\hat{n} = \hat{a}^\dagger \hat{a}$) and the uncertainty of this value (Δn) in a coherent state $|\alpha\rangle$.
- One can define "reduced" position and momentum operators

$$\hat{Q} = \frac{1}{\sqrt{2}} \left(\hat{a} + \hat{a}^\dagger \right)$$
$$\hat{P} = -\frac{i}{\sqrt{2}} \left(\hat{a} - \hat{a}^\dagger \right).$$

  Show that $(\Delta P)^2 (\Delta Q)^2 = \frac{1}{4}$ for a coherent state $|\alpha\rangle$. What does this mean in relation to the uncertainty principle?

3. The Λ particle is a neutral baryon with mass $M \simeq 1115$ MeV/c^2, which after an average lifetime of $\tau = 2.9 \times 10^{-10}$ s decays into a nucleon of mass $m_1 \simeq 939$ MeV/c^2 and a π meson of mass $m_2 \simeq 140$ MeV/c^2. The decay $\Lambda \to p + \pi^-$ was first observed in a cloud chamber. The tracks from the decay particles arise from a single point and have the appearance of a V or a Λ. The identity and momentum of the particles can be determined from their range or the radius of curvature of their path in the magnetic field of the chamber.

   (a) Express M^2 as a function of m_1, m_2, the magnitudes of the momenta of the decay particles p_1, p_2 and the opening angle θ between the two tracks. Use the conservation of energy and momentum when deriving the expression.

   (b) A Λ particle of total energy 10 GeV is created in a collision process in the upper plate of a cloud chamber. What is the average flight distance within the chamber before it disintegrates? *Hint: pay attention to time dilation or length contraction.*